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Technical Notes

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Effectiveness of Complex Design Through an Evolutionary Approach

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Nomenclature

A	=	area, m^2
k	=	thermal conductivity, $W \cdot m^{-1} \cdot k^{-1}$
L	=	length, m
n_{bif}	=	number of bifurcations
n_{gen}	=	number of generations
n_{ind}	=	number of individuals
q'''	=	internal heat generation, $W \cdot m^{-3}$
$\dot{S}'_{\text{gen}}$	=	entropy generation, $W \cdot m^{-1} \cdot k^{-1}$
T	=	temperature, K
t	=	thickness, n
ε	=	dimensionless thermal conductivity, k_0/k_f
ϕ_f	=	area ratio, A_f/A

Superscripts

*	=	dimensionless variable

I. Introduction

ONE of the apparent paradoxes of nature is the appearance of niches of structural complexity in a universe that is inexorably moving toward greater total entropy. The development of low entropy generation structures seems to contradict what is expected from a universe that is driven by the second law of thermodynamics. However, the second law itself hints on the mechanism that drives such structures to be formed, and also explains why they thrive as islands of low-entropy generation in disordered surroundings. The notion that reversible processes are intrinsically more efficient (combined with the energy conservation of the first law) creates a driving mechanism to extract energy from energy potentials (or gradients) by means of minimum entropy generation. Thus, any evolutionary or competitive process will select potential-depleting structures that, although contributing to the generation of total entropy, are themselves highly organized low-entropy generating structures. In this work, we use a simple conduction heat transfer problem to illustrate the complex interaction between the first two laws of thermodynamics and this new principle of competitive pursuit of complexity driven by low-entropy generation.

There are numerous examples in which complexity is an intrinsic part of evolutionary design (e.g., cardiovascular systems, tree leaves, insect wings, etc.) [1–3]. In this context, several noticeable features such as multiple bifurcations and branches emerge in a variety of systems and applications. Perhaps even more interesting is the observation that what allows a species or a particular structural arrangement to survive a competitive selection process is its general level of complexity, and not its detailed features. In other words, beyond a given complexity level, the specific details of the energy networks are not relevant, and the robustness of the system complexity become evident. Thus, the vascular systems of different individuals within a species are not identical, but they share similar sizes, distributions, and complexity levels. The complexity level is a common feature in each one of the fittest structural arrangements that, regardless of the applications for which they were evolved, allow them to survive under a wide range of conditions.

The advantages associated with dendritic structures in natural systems were addressed in previous studies [1–3]. Universal optimal geometrical features have been identified as a result of studies [4–11]. Recently, a series of studies employed evolutionary processes similar to those observed in nature to propose a new generation of high-performance engineering devices [12–14]. In particular, remarkable contributions have being made using constrained constructive optimization (CCO) [15], in which three-dimensional computational models generated optimized fluid distribution networks [15–19], and through constructal theory [12–14]. The general problem of determining optimal dendritic structures for heat and mass transfer can also be cast as a topology optimization problem, for which many effective methods exist in the literature [20–22]. The key feature of such designs is their hierarchical complexity [1–22]. In other words, the ability of a system to collect or deliver energy (or mass, goods, etc.) to an area or volume in a nearly uniform way.

The objective of the present work is to show the effectiveness and the robustness of added complexity of a family of trees through the use of an evolutionary optimization algorithm [23–25]. The method is used to illustrate the solution process for an area-to-point-type heat transfer problem [12–15,26].

II. Physical Problem

As originally proposed in [12–14,26], consider the area A in Fig. 1, which is filled with an isotropic solid of conductivity k_0 that generates heat at a uniform rate q'''. All external boundaries of the area A are insulated ($\partial T/\partial n = 0$), except by a conductive strip of width t_0 located on the center of the left wall, which is maintained at

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T_0. There is a fixed amount of solid material of conductivity k_f ($\varepsilon = k_0/k_f \ll 1$) such that $\phi_f = A_f/A(\phi_f < 1)$. The highly conductive material is connected to the heat sink at T_0. Assuming continuity ($k_0 \partial T/\partial n = k_f \partial T/\partial n$) on the interface between the heat generating body and the highly conductive material, the problem of optimal distribution of the conductive material is reduced to finding the optimal dendritic structure among a family of trees, such that the maximum temperature in A is minimized. This is equivalent to the minimization of work lost by a hypothetical heat engine that operates between a finite temperature gap (i.e., $T_{\max} - T_0$) and it is subjected to a given heat flow current ($\dot{Q}' = q'''A$). Application of the second law [12] to this problem yields

$$\dot{S}'_{\text{gen}} = \dot{Q}'\left(\frac{1}{T_0} - \frac{1}{T_{\max}}\right), \qquad \dot{Q}' > 0; \qquad T_{\max} \geq T_0 \tag{1}$$

which shows that minimizing $T_{\max}$ is equivalent to the minimization of $\dot{S}'_{\text{gen}}$.

The family of trees used in the optimization is characterized by branches that bifurcate at angles α_i after stretching a length L_i from a junction (see Fig. 1). The length and thickness of each branch can decrease after each bifurcation at a given ratio ρ. The initial length and thickness of the branches, as well as their bifurcation angle and ratio of thinning and shortening are the four parameters to be optimized.

A. Mathematical Formulation

Because the distribution of the cooling path over A is expected to yield complex geometries, no analytical method is suitable for solving the stated problem. Thus, a two-dimensional numerical solution method was used. The numerical simulations consist of solving the linear system of equations $\nabla^2 T + \xi = 0$ for two domains characterized by either $\xi = q'''$ or $\xi = 0$, which correspond to the heat generating area and the highly conductive path, respectively.

Our numerical simulations were performed using the finite elements method toolbox COMSOL Multiphysics, version 3.2 from COMSOL, Inc., using Lagrange quadratic finite elements. The mesh density was extensively tested to guarantee that the results were grid independent. These tests, which are omitted for brevity, showed that the dimensionless temperature, defined as $T^* = (T - T_0)/(q'''A/k_0)$, reached an acceptable level of convergence (i.e., the dimension temperature varied less than 0.1% if the mesh was doubled) when the number of elements is between 10,000 and 40,000, depending weakly on the complexity of the cooling tree.

B. Optimization Method

The optimization of the dendritic structure was performed using a script developed for the genetic algorithm (GA) toolbox from MATLAB. This method is suitable for an evolutionary optimization of multi-independent variables problems, as needed in the solution procedure that we propose. The dimensionless fitness function to be minimized is the maximum dimensionless temperature $T^*_{\max} = (T_{\max} - T_0)/(q'''A/k_0)$. The evolution of the designs used two genetic operators: crossover and mutation. The crossover acts on 80% of the population. Parental selection is based on the stochastic uniform method, which means that the contribution of each parent is proportional to the value of its fitness function. The gene recombination is performed with the scattered method, in which a random binary vector defines which genes come from the first parent (those labeled 1) and from the second parent (labeled 0). Finally, a Gaussian mutation with shrinking variance was selected for the mutation operator.

In the GA, an initial population of 100 individuals n_{ind} was used. The population was generally allowed to evolve for 200 generations n_{gen}, which was large enough to guarantee the convergence between the *fittest individuals* and the average value of the fitness function of the whole population. The complexity level of each individual is measured through the number of bifurcations n_{bif} [12–14,26]. Five different complexity levels were considered (i.e., $i = 0$, 1, 2, 3, and 4). In the simplest configuration, no bifurcation was allowed and the optimal design is the one in which the fin has a length L and a thickness $t_0 = A\Phi_f/L$. The performance of this configuration was used as a benchmark. In addition to the number of bifurcations, each configuration had four degrees of freedom: 1) the bifurcation angle ($5 < \alpha_i < 95$ deg), 2) the length of the first branch L_0 ($0 < L_0 < L$), 3) the thickness of the first branch t_0 ($0 < t_0 < A\Phi_f/L$), and 4) the reduction ratio between successive branches $\rho = t_{i+1}/t_i = L_{i+1}/L_i$ ($0.25 < \rho < 1$). These four parameters were entered in the GA as independent variables. The algorithm searches values of the independent variables within the given limits in search of the best topology. The computational time required to evolve a given population of complex individuals varied between 25 ($i = 4$) and 50 ($i = 4$) generations per day, depending on the topology of the dendritic structure. The calculations were performed in a Pentium 4 with 3.2 GHz of clock speed and 2 Gb of RAM memory. Note, however, that the hardware industry is moving toward massive parallel architectures [27], and that genetic algorithms, being inherently parallel in formulation, are naturally suited for such architectures and form an attractive class of optimization methods for the next generation of massive parallel computers.

III. Discussion

Figure 2 shows the evolution of the fitness function ($T^*_{\max,4}|_{\min}$) of the fittest individuals with the number of generations for a structure with four bifurcation levels. According to Fig. 2, a small number of generations provide a significant improvement in the performance of the fittest individuals. More specifically, after 40 generations, the fitness function varied from 0.57159 to 0.32374. The direct effect of such evolution is given through the six color frames, which show not only the tree design, but also the temperature distribution on area A within a single dimensionless temperature scale. Also interesting is the evolution of the design of the fittest individuals. In the first and second generations, the canopy of the dendritic structure is highly concentrated at the center of A and, as a result, undesired hot temperature spots appear in all corners. As the population evolves,

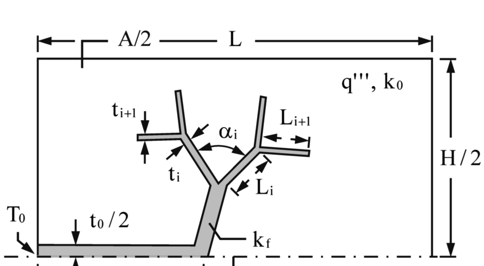

Fig. 1 Physical domain.

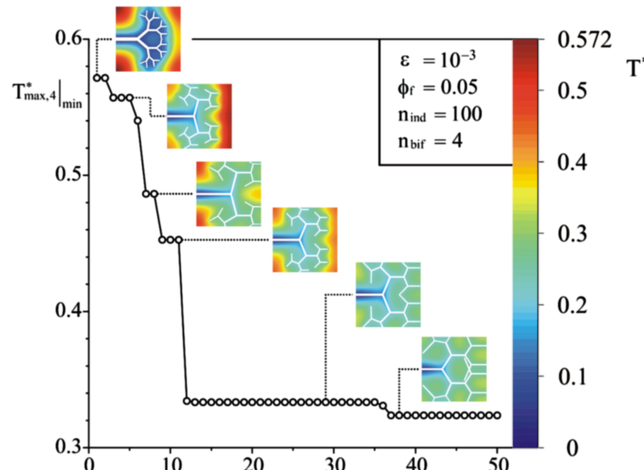

Fig. 2 Evolution of the elite individuals with the number of generations.

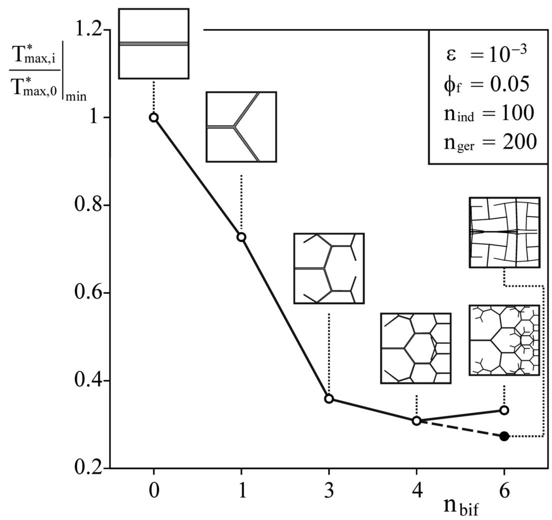

Fig. 3 Effect of the complexity on the fitness function $T^*_{max}|_{min}$.

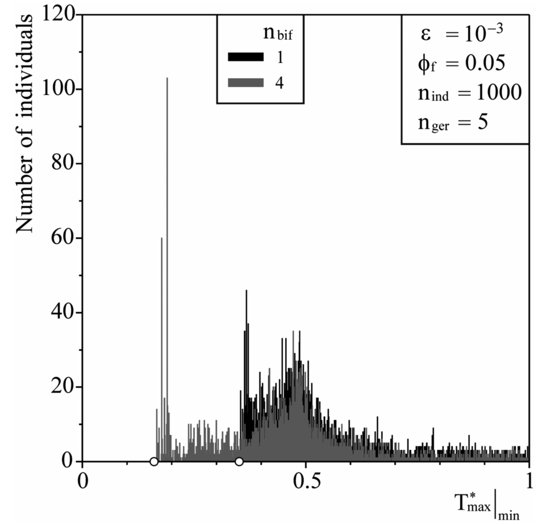

Fig. 4 Efficiency and robustness of evolved populations of individuals with different complexity levels.

the GA algorithm searches for designs that minimize the hot spot temperatures. Consequently, the evolutionary process arrives at designs that attempt to populate all regions of A with cooling branches. A caveat of this approach should be mentioned here: because the root of the dendritic structure, which is maintained at $T^* = 0$, is located on the left-hand side, fewer branches are needed in the left half of the domain as compared with the right half. This "imperfection" of the evolutionary process is no different than the restriction on contiguous growth of living tissue that restricts the development of living organisms.

Although no analytical solution for this kind of problem exists, one limiting solution is known. This limiting solution refers to the case of $\varepsilon \to 0$, where the temperature differential between the root of the cooling path and its extremes approaches zero, and therefore the problem is reduced to a geometrical optimization problem. For a limited amount of material, the solution of this maxima and minima problem is a honeycomb distribution, because the hexagonal honeycomb geometry maximizes the internal area for the minimum amount of walling. It is noteworthy to mention that the GA approaches this solution within 40–50 generations.

Figure 3 shows the effect of the optimized complexity on the normalized fitness function. The simplest design possible is the one with zero bifurcations ($n_{bif} = 0$). In this case, the optimal design is a rectangular fin through the symmetry plane. The fitness function value for such configuration is $T^*_{max,0}|_{min} = 0.482551$. As the dendritic structure evolves, substantial improvement in the value of the normalized fitness function occurs. Our results show that when one bifurcation is added to the system ($n_{bif} = 1$), the normalized fitness function drops to 70% of its original value. Further improvements are obtained as the complexity of the dendritic structure increases. For the given family of fittest individuals shown in Fig. 3 (open symbols), the optimal complexity level happens when $n_{bif} = 4$, where reductions larger than 70% are observed in the fitness function. Also interesting is the emergence of a new family of individuals with six bifurcation levels (solid symbol), where the normalized fitness function was reduced by 73% when compared with the simplest design ($i = 0$). It is also important to note the fact that the optimal design shown in Fig. 3 is not necessarily the overall best topological design possible; it is conceivable that other distributions of thermal conductivity would lead to better performance. Instead, the results shown correspond to the best designs for different degrees of complexity within the family of trees described in Sec. II.B.

In Fig. 4, we further explore the efficiency of complex design by comparing the combined frequency of fittest individuals with two different number of bifurcations (i.e., $n_{bif} = 0$ and 4). For this analysis, a thousand randomly generated individuals were allowed to evolve for five generations. The open symbols located in the abscissa refer to the optimized design shown previously in Fig. 3 for configurations where $n_{bif} = 1$ and 4. The main conclusion drawn from Fig. 4 is that the complex structures not only provide better designs, but these designs are robust to perturbations too. Indeed, any design for which $n_{bif} = 4$ and with a fitness function value located between the two open symbols performs better than the best fully optimized individual with a single bifurcation. This is a clear argument in favor of complexity as a robust feature of design.

IV. Conclusions

The GA evolutionary process described in this work shows that the complexity level can be an important parameter in design, more so than the specific features of the branched structures. We also show that the importance of providing a fully optimized configuration became less relevant as complexity is added to the structure. This is an important finding because it highlights the robustness of added complexity, a factor that becomes a survival determinant when fluctuations in environmental conditions are considered. The robustness of complexity in shape optimization, which is here supported quantitatively by our GA calculations, is in agreement with previous works in the field that were not based on genetic optimization (e.g., [12–14,26]). The less defined optimal design for increased complexity explains the differences observed between complex natural individuals (i.e., systems) of a same species, which are able to thrive due to their hierarchical complex structural patterns, even though no individual in the species is identical. The conclusion is that if a threshold level of complexity is attained by a design, it will perform with nearly optimal efficiency because of the robustness granted by its complexity, regardless of the smaller details of its structure.

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